Long-time asymptotics for the defocusing integrable discrete nonlinear Schrödinger equation

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Abstract:

We study the long-time behavior of the defocusing integrable discrete nonlinear Schrödinger equation introduced by Ablowitz-Ladik on the doubly infinite lattice (i.e. $n \in \mathbf{Z}$)

$$i\frac{d}{dt}R_n + (R_{n+1} - 2R_n + R_{n-1}) - |R_n|^2(R_{n+1} + R_{n-1}) = 0. \tag{1}$$

It is an integrable discretization of

$$iu_t + u_{xx} - 2u|u|^2 = 0. \tag{2}$$

It is known that (1) is the compatibility condition of the following AKNS pair:

$$\begin{aligned} X_{n+1} &= \begin{bmatrix} z & \bar{R}_n \\ R_n & z^{-1} \end{bmatrix} X_n, \\ \frac{d}{dt}X_n &= \begin{bmatrix} iR_{n-1}\bar{R}_n - \frac{i}{2}(z - z^{-1})^2 & -i(z\bar{R}_n - z^{-1}\bar{R}_{n-1}) \\ i(z^{-1}R_n - zR_{n-1}) & -iR_n\bar{R}_{n-1} + \frac{i}{2}(z - z^{-1})^2 \end{bmatrix} X_n. \end{aligned}$$

We have obtained the long-time asymptotics of (1) by using the nonlinear steepest descent method of Deift-Zhou. Roughly speaking, the result is as follows. If $|n/t| < 2$, there exist $p_j = p_j(n/t), q_j = q_j(n/t) \in \mathbf{R}$ and $C_j = C_j(n/t) \in \mathbf{C}$ $(j = 1, 2)$ such that

$$R_n(t) = \sum_{j=1}^{2} C_j t^{-1/2} e^{-i(p_j t + q_j \log t)} + O(t^{-1}\log t) \quad \text{as } t \to \infty. \tag{3}$$

The quantities q_j and C_j are defined in terms of *the reflection coefficient* that is associated with the potential $\{R_n(0)\}_n$. Each term in the sum exhibits the behavior of *decaying oscillation* of order $t^{-1/2}$. Notice that in the case of (2) the asymptotic behavior is expressed by a single term.

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